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Searching for the P/E Mean Reversion Affinity – An Application of the Flexible Fourier Approximation

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The S&P 500's Price/Earning mean reversion phenomenon seriously threatens foundations of the efficient market hypothesis. Using the Fourier approximation and regime switching models, the P/E mean reversion issue has been further investigated. The empirical findings of the threshold autoregressive model during 1871:12–2016:3, suggest that the P/E mean reversion tendency can be justified only in an economic expansion during which the P/E ratio stays afloat above its likely long-run threshold. However, the speed of adjustment toward the historical long-run equilibrium is practically non-existent in contractionary periods during which the P/E ratio tends to be below its estimated threshold.

Keywords: P/E Ratio, Mean Reversion, Fourier Approximation, TAR and MTAR Models, Speed of Adjustments

JEL Classification: C4, G1

I. Introduction

It has been argued that the monthly stock price index proxied by the S&P 500's (P)/12-month average returns (E), the so-called P/E ratio, can be utilized to predict the future movements of P and E. If so, it implies the existence of mean reversion for P/E, which in and of itself contradicts the efficient market hypothesis (EMH). The EMH asserts that the stock price is sensitive to the minuscule newly emerging information in the market and that the movements of stock prices are volatile and unpredictable. Accordingly, there should not be a momentous discrepancy between the optimal forecast and actual (equilibrium) stock prices, so that the likelihood of making abnormal profits in the stock market is asymptotically zero.

In the finance and macroeconomics literature, the P/E mean reversion issue has been investigated extensively, and the empirical evidence for and against such a tendency has been presented. For example, Campbell and Shiller (1988) contend that if the mean reversion makes a brief appearance unexpectedly, the valuation ratios, such as P/E and the dividend-price ratio, fluctuate back and forth in a valuation tunnel. Eventually though, when such ratios reach an exceptionally high/low value, any lopsided move should not last long and the market fundamentals bring these ratios back into the normal range. In other words, the valuation ratios are apt to remain stable around their corresponding historical mean values – especially the P/E ratio with the embedded mean reversion feature. Campbell and Shiller's argument (among others) has been used as evidence that the stock market may not be fully efficient.

More recently, Becker *et al*. (2012) state that the existence of non-linear stationarity in the P/E ratio time series is also able to substantiate the mean reversion, implying that the P/E ratio

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gradually returns to the historical equilibrium mean value. Implementing the Fourier unit root test, they robustly reject the non-stationarity of the P/E ratio, while observing downturns tracked by the sine function term are negligible and statistically insignificant.[1] Moreover, in order to authenticate their Fourier's findings, they use the Markov switching model in which the P/E time series switches from one regime into another depending on the conditionally estimated probability of each. By changing the frequency of the P/E data (3-month/3-month average, 6-month/6-month average, and 12-month/12-month average), they estimate six Markov models. The authors conclude that the transitional probability of P/E residing in regime 1 (P11, the default regime) in which the P/E valuation metric is stationary, is much higher than in regime 2 (P22, transitory regime) in which P/E is non-stationary.[2] As such, the Markov model's findings that assume structural changes are generally stochastic are quite in line with those of the Fourier trigonometric approximation that presume such changes are mainly deterministic. Moreover, their computed recurring cycles in the 1881-2003 sample period is 3.7, and the interval between two consecutive cycles crossing the long-run equilibrium mean values is 33-years. Based on this conclusion, investors can follow the P/E movements, make a prediction of the P/E ratio, and further forecast the stock price index. Their findings suggest that structural breaks with recursive occurrence play a major role in directing the apparent stationarity of the P/E ratio. The advantage of this approach is that there is no need to figure out how many structural breakpoints have been embedded, where exact breakpoints have been located, or what pattern the P/E series has taken - linearly or cyclically. Most importantly, allowing for unknown structural breakpoints is vital for rendering the conclusion of stationarity entrenched in the P/E ratio time series, implying that an increase in the P/E ratio should be followed by either lower P or higher E.

On the other hand, there are a number of researchers who steadfastly argue for the P/E mean aversion propensity – see for instance Glassman and Hassett (2000), and Elias (1999)[3]. Their key argument is that the P/E ratio inherits a great deal of non-stationarity from the stock price random walk tendency and, as a consequence, its predictability power is markedly poor. Clouding this matter even further, the most recent financial market experience provides an added support for the alleged stock market inefficiency. The market started plunging drastically in 2008 after the stock price climbing high up to the crest stage in 2007:10, followed by an abrupt decline in 2009:03. The overwhelmingly unexpected losses, in the stock market in particular and in capital markets in general, provide a picture of inefficiency in their operations.

To validate the stationarity of a time series, researchers have made a great deal of progress on tests for structural changes, ranging from a single break to multiple breakpoints, and from level break(s) to non-linear break(s). With that in mind, the econometric scope of some of these tests is quite limited in that some cannot capture the characteristic of a series with more than one or two breaks, see Perron (1989), Lee and Strazicich (2003). Some, as referred to by Prodan (2008), are not capable of predicting the series without knowing the exact number, locations, and magnitude of multiple breaks. However, Enders and Lee (2012) present a variant of the Fourier approximation

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[1] They also utilize the ADF, along with TAR and MTAR unit root tests for which a threshold of zero has been hypothesized.

[2] Note that the main difference between the Markov and TAR models revolves around these estimated conditional probabilities. As such, if P11 > P22 substantially, then regime 1 is quite dominant – and thus, the estimates for regime 2 are questionable. Furthermore, the degree of persistence is also affected by the estimated auto-regression coefficients (ρ) in each regime in that if $\rho 1 > \rho 2$ and P11 > P22, the process is mainly trapped in regime 1 and regime 2 is quite irrelevant. Finally, the unconditional probabilities of being in each regime (P1 and P2, respectively), are also affected because for regime 1, P1 = (1-P22)/ (2 – P11 – P22).

[3] See Becker *et al.* (2012) for more information.

to account for both the unknown structural breakpoints, and a non-standard F-test for linearity – provided that there is no residual autocorrelation in the approximation process. The Fourier test utilizes a dynamic (time variant) deterministic intercept term, consisted of sine and cosine functions to grasp the essence of the process, no matter what the global pattern of a variable is, or whether there is a breakpoint/non-linear trend. They focus on the specific data-generating regression model with the smallest sum of squared residual at the most appropriate frequency, as well as a more precise approximation including multiple (cumulative) frequencies.

If the P/E ratio shows the characteristic of non-linearity, it is quite reasonable to inquire whether or not the series is still stationary and mean reverting. If so, how long should it take for this valuation metric to return to its historical mean value? To provide a viable answer to the above inquiry and to grasp the essence of market efficiency, this paper expands the work of Becker *et al.* (2012). Most importantly, if the P/E ratio follows a non-random walk in an expanded sample using an entirely different regime switching environment, at what speed of adjustment does it move toward its long-run unconditional mean? If the estimated speed of adjustment for this valuation ratio is substantial and significantly different from zero at a sensible significance level, then the mean reversion theory is confirmed. However, if the adjustment process is sluggish and insignificant, that might imply the P/E mean aversion phenomenon. As such, the main contribution of this research is the addition of the Threshold Auto-Regressive (TAR) and Momentum Threshold Auto-Regressive (MTAR) models, which enable estimating the speed of adjustments not only for the P/E ratio, but also for its components (P & E). It is widely believed that P tends to be volatile (non-stationary), while E is relatively tranquil (stationary) and more predictable. Ultimately, whether P/E is mean reverting or averting is mainly determined by the dominant trait of its component. Toward that end, the empirical findings are presented in Section II. The threshold modeling along with related findings are discussed in Section III, followed by concluding remarks in Section IV.

II. Empirical Findings

To follow up on the research of Becker *et al*. (2012), the monthly data are obtained from Shiller's (2016) website. The P/E ratio is a monthly time series, extended for about 14 years starting from December 1871 to March 2016. The P/E time series has been computed by dividing the monthly averages of daily stock price index (S&P 500's) by the 12-month moving average of composite earnings. As is common in this literature (i.e., as a stepping stone), we start by using the traditional Augmented Dickey Fuller (ADF – 1979) unit root test to examine the stationarity of the P/E ratio as follows.

$$\Delta(P/E)_t = \alpha_1 + \beta_1 (P/E)_{t-1} + \beta_2 \text{ Trend} + \theta_i \Delta(P/E)_{t-i} + \varepsilon_{1t} \qquad (1)$$
$$i = 1, 2, 3, \ldots,$$

where Δ is the first differencing operator, α_1 is the intercept, Trend is a deterministic linear time trend, β_1, β_2, and θ_i are the regression coefficients, and ε_1 is a white noise error term. If the P/E ratio time series data are stationary (a necessary condition for the P/E mean reversion), then the estimated *t*-statistics for β_1 should be significantly larger than the Dickey-Fuller τ-value. The findings are presented in Table 1.

Table 1: The ADF Unit Root Test

Variable	Coefficient	*t*-statistics	Probability
α_1	0.2721	4.8605	0.0000
$(P/E)_{t-1}$	-0.0240	-7.0082	0.0000
$\Delta(P/E)_{t-1}$	0.6733	28.4006	0.0000
$\Delta(P/E)_{t-2}$	-0.0155	-0.5397	0.5894
$\Delta(P/E)_{t-3}$	-0.0538	-2.2333	0.0257
TREND	0.0001	2.8137	0.0050
R^2	0.4248		
Q-statistics	3.16 (Probability = 0.2063)		

Note: The critical τ–values are -3.41 and -3.96 at the 5 and 1 percent significance levels, respectively.

As can be seen, the null hypothesis of non-stationarity for the P/E ratio is resoundingly rejected at any significance level. Moreover, the Breusch-Godfrey Lagrange Multiplier test (Q-statistics) indicates that there is no significant evidence of residual autocorrelation. However, the ADF unit root test is deficient in that its dynamic adjustments are predominantly linear, and thus it is incapable of dealing with potential breakpoints in the P/E time series. Luckily, there is a way to tackle a single-breakpoint problem if we can pinpoint its exact location. That is, by splitting the sample dataset into sub-periods, one can observe whether the P/E ratio moves in a stable way or otherwise in each of these sub-periods. For example, the P/E ratio reached the historical high value of over 80 in October 2009, which is an appropriate breakpoint within the sample period. To explore this approach further, the time series data can be divided into two smaller samples: 1871:12-2009.10 and 2009.11-2016.03. Then, a dichotomous dummy variable can be established to represent the lift change that can be incorporated into the ADF unit root test. However, since the known breakpoints are found by a visual observation rather than a formal statistical testing, two questions are warranted. First, are there other structural breaks in the time series besides those detected? Secondly, are those moving patterns containing breakpoints behaving in the form of gradual or abrupt structural breaks?

To deal with the above inquiries, as proposed by Enders and Lee (2012), the flexible Fourier approximation is an appropriate and versatile mechanism. Most notably, the dominant feature of this test is a deterministic intercept term, including sine and cosine functions which are capable of not merely keeping track of the non-linear cyclical changes, but also the structural breakpoints. Substituting for $\alpha_1 = \alpha_t$ its Fourier's approximation results in:

$$\Delta(P/E)_t = [\alpha_0 + \sum_{k=0}^{n} \mu(k) \sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=0}^{n} \nu(k) \cos\left(\frac{2\pi kt}{T}\right)] + \beta_3 (P/E)_{t-1} + \beta_4 \text{ Trend} + \theta_{1i} \Delta(P/E)_{t-i} + \varepsilon_2. \quad (2)$$

In Equation (2), the term in square brackets is the deterministic intercept (α_t), which is a function of time (t), n represents the number of recursive frequencies, k is a specific frequency, T is the number of observations, β_3 - β_4, μ, and ν are the regression coefficients to be estimated, and ε_2 is a classic error term. The sine function terms are for keeping track of abrupt falling breakpoints or the cyclical downturns, while the cosine function is focusing on the sudden uplifting break changes or gradual upturns.

Suppose there are no breakpoints detected in which case the coefficients of sine and cosine terms are zero, i.e., $\mu(k) = \nu(k) = \cdots = 0$ and thus, the conventional ADF-test would be powerful enough to check the stationarity proposition. However, if there are some certain types of breaks, either abrupt or cyclical, then at least one of the sine or cosine functions is different from zero. By practicing trial-and-error experiments, in order to capture the structural changes and patterns at the optimal frequency, we use a single-frequency testing regression model. Subsequently, by looking at the statistical characteristic of k = 1 to k = 5 models, it has been concluded that k = 4 is the optimal one, yielding the smallest sum of squared residuals (SSR). The findings are summarized in the first two columns of Table 2.

Table 2: Fourier's Unit Root Tests

Explanatory Variables	**Optimal Frequency (k=4)**		**Cumulative Frequencies**	
	Coefficient	***t*-statistics**	**Coefficient**	***t*-statistics**
Intercept	0.3675	5.7085	0.6001	4.9512
$\Delta(P/E)_{t-1}$	-0.0311	-7.4845	-0.0462	-9.1971
TREND	0.0001	3.1421	0.0001	1.2657
sin(8πt/T)	-0.0665	-2.1475	-0.1179	-3.3181
cos(8πt/T)	-0.1081	-3.5243	-0.1509	-4.7842
sin(6πt/T)	-	-	-0.0858	-2.3667
cos(6πt/T)	-	-	-0.0427	-1.5139
sin(4πt/T)	-	-	-0.0355	-0.8461
cos(4πt/T)	-	-	0.0526	1.8444
sin(2πt/T)	-	-	0.0009	-0.0145
cos(2πt/T)	-	-	0.1438	4.5111
Augmentation terms - omitted	6		6	
Sum of squared residuals (SSR)	1165.033		1146.218	
Akaike Information Criterion (AIC)	2.458		2.448	
Q-statistics	2.06 (Probability = 0.36)		1.71 (Probability = 0.43)	
Linearity Test:	$F_k = 7.55$			

Notes: For the single-frequency Fourier unit root test, the critical *t*-values are -3.63 and -4.24, whereas for cumulative frequencies they are -6.05 and -6.57 at the significance level of 5 percent and 1 percent, respectively. For the Q-test, the optimum lagged residual terms have been determined by minimizing the AIC. The critical value of $F(\check{k})$ for the linearity test with a sample size of 2500 is 7.50 at the 1 percent significance level. See Enders and Lee (2012).

Unmistakably, we can reject the null hypothesis of a unit root for P/E at any significance level due to the fact that *t*-statistics = -7.4845. The coefficient of -0.0311 implies that the current $\Delta(P/E)_t$ is negatively related to the previous moving direction – and eventually becomes negligible in a waving pattern. Both the sine and cosine functions are statistically significant at the 5 percent level. This illustrates that there are four non-linear cycles embedded in the P/E ratio time series. Furthermore, the linearity test delves into the idea that those 4-optimal breaks are non-linear breakpoints. The estimated $F_k = 7.55$ is larger than the corresponding critical value. This feature concurs with the substantial upward trend, which is determined by the coefficient of trend 0.0001 and its respective significant *t*-statistics. Finally, when the P/E ratio sample period lasts over

145 years, it would take the series about 35 years to repeat itself, which is slightly longer than that of Becker *et al*. (2012) (33-year-long cycles).[4]

The results with multiple frequencies shown in the last two columns of Table 2 also profoundly reject the non-stationarity of the P/E ratio. The sine and cosine functions for k = 1, 2, and 3 are added to the previous regression model. As demonstrated, the significant *t*-statistics for the sine function (k = 3 and 4) and cosine functions (k = 1, 2, and 4) are helpful in interpreting the behavior of the P/E ratio. Similar to the optimal single-frequency Fourier test (k = 4), the cumulative approximation process also attaches importance to those five consecutive patterns. In addition to the five significant frequent cycles, there are a handful of less frequent motion curves. Most notably, both the mono-chronic (k = 1) and bi-chronic (k = 2) moving cycles are energetic recovering upturns, resulting from the estimated cosine terms for k = 1, k = 2, and significant *t*-statistics (4.5111 and 1.8444, respectively). Consistent with the findings on moving patterns by the optimal single-frequency Fourier model, the cumulative model is able to detect those five more frequent and three infrequent breakpoints as non-linear abrupt lift changes, rather than linear or gradual moves. Even though both single frequency and cumulative models are able to grasp the stationarity of the P/E ratio, the latter has much improvement in reducing the variation of SSR and the AIC. As such, there appears to be adequate evidence to surmise that the versatile multiple frequencies regression model is more practical in testing the stationarity of the P/E ratio. Lastly, in both approximations, the underlying autocorrelation issues have been dealt with by adding appropriate augmentation terms.

III. Regime Switching Speed of Adjustments

In accordance with the estimated cumulative Fourier approximation, the P/E ratio is a stationary process with non-linear speed of adjustments and a realized insignificant linear uptrend. Wherever the current P/E ratio is, it would be inevitably returning to the unconditional historical mean. However, even the multiple frequencies model does not have the competency to assert how soon the P/E ratio will "hit the home runs," reaching its equilibrium value in the long run. Consequently, in this section, an outlet has been introduced in order to gauge the speed at which the P/E ratio moves toward its historical unconditional mean.[5] The outlet mainly combines the Non-linear Error Correction Modeling (NLECM) with partitioning of the P/E time series relative to its threshold. Subsequently, the estimated NLECM enables us to comprehend the behavior of P/E in different domains, and the mechanism by which it approaches the long-run destination. The threshold here is the long-run equilibrium (unconditional mean), splitting the P/E ratio into the higher-value (expansionary) regime and the lower-value (recessionary) regime as depicted by model (3).

$$\Delta(P/E)_t = \rho_1 \, IND \, [(P/E)_{t-1} - \Gamma)] + \rho_2 \, (1\text{-}IND) \, [(P/E)_{t-1} - \Gamma)] + \sum \theta 2_i \, [\Delta(P/E)_{t-i}] + \varepsilon_3 \quad (3)$$
$$i = 1, 2, 3, \ldots,$$

[4] The findings are available upon request.

[5] As a side note, both P and E are integrated of order one and in line with the Engle/Granger theorem, a linear combination of them should be co-integrated. Indeed, both the Engle and Granger (1987) and Johansen (1995) tests depict a co-integrating vector between these two variables and thus, one can be considered as a rational forecast of the other. The empirical findings are available upon request. See also Stock and Watson (1993), Elliott *et al*. (1996), and MacKinnon (1996).

where Γ is the long-run equilibrium (threshold) for the P/E ratio, ρ_1 and ρ_2 are the auto-regression coefficients depicting the speed at which P/E adjusts to its long-run equilibrium (given the threshold, Γ). Moreover, the augmentation term $\sum(\theta 2)$ tackles the autocorrelation problem, IND is an indicator that identifies whether the P/E ratio is in the higher-value scenario (generally, indicative of a prosperous economy), while (1-IND) is correspondingly a potential recessionary identifier. The specified Heavyside indicator functions are IND = 1 if $(P/E)_{t-1} \geq \Gamma$, and 0 otherwise [i.e., $(P/E)_{t-1} < \Gamma$] for TAR, and IND = 1 if $\Delta(P/E)_{t-1} > 0$, and 0 otherwise [i.e., $\Delta(P/E)_{t-1} \leq 0$] for MTAR. Assuming the existence of an attractor by rejecting the null hypothesis that $\rho_1 = \rho_2 = 0$ (based on the non-standard F-test), rejecting $\rho_1 = \rho_2$ (based on the standard F-test) is indicative of non-linear (asymmetric) dynamic adjustments. In essence, the NLECM is a logical generalization of Equation (3) by way of incorporating appropriate lagged values of both the dependent and independent variables.[6]

The numerical value of Γ would have to be estimated in the same way as the numerical values of ρ_1 and ρ_2. A consistent estimate of Γ has been obtained in accordance with the procedure explicated by Chan (1993). The Chan approach precludes ± 15 percent of the observations and also ranks them in an ascending fashion. Moreover, using OLS, Equation (3) has been estimated recursively within ± 15 percent constraint. The estimated model whose sum of squared residual is minimal produces a consistent estimate of Γ, which can be used to estimate Equation (3) suitably.[7]

The NLECMs for the ingredient of the P/E ratio are also established separately. The idea is to find out how quickly/slowly the factoring variables move toward their own equilibrium points in the two pre-determined regimes. The relative movement of the numerator (P) compared to that of the denominator (E), would eventually determine how long it would take for P/E to reach the intended destination. The coherence in the pace of P and E drives the P/E ratio to persist, while irrational volatility and inconsistency of the component would render an unstable P/E in the long run. In practice, these two ingredient regressions make use of the same Heavyside indicators as those of the P/E ratio. The NLECMs are specified below.

$$\Delta P_t = \rho_3 \, IND \, [(P/E)_{t-1} - \Gamma] + \rho_4 \, (1\text{-}IND) \, [(P/E)_{t-1} - \Gamma] + \sum \delta_i \, \Delta P_{t-i} + \sum \varphi_i \, \Delta E_{t-i} + \varepsilon_{t4} \quad (4)$$

$$\Delta E_t = \rho_5 \, IND \, [(P/E)_{t-1} - \Gamma] + \rho_6 \, (1\text{-}IND) \, [(P/E)_{t-1} - \Gamma] + \sum \delta 1_i \, \Delta P_{t-i} + \sum \varphi 1_i \, \Delta E_{t-i} + \varepsilon_{t5} \quad (5)$$

$$i = 1, 2, 3, \ldots,$$

where $\rho_3 - \rho_6$ are the speed of adjustment parameters, δ_i and φ_i are the augmentation term coefficients, and ε_4 and ε_5 are white noise error terms. The Chan (1993) estimation procedure provides the conditional mean (Γ = 19.70) with the smallest sum of squared residual for TAR. The estimated threshold makes it possible to explore the moving pattern of the P/E ratio in the aforementioned two domains. The findings are reported in the upper portion of Table 3.

[6] For more information, see Enders and Granger (1998).
[7] The detailed estimation and findings are available upon request.

Table 3: Empirical Findings of the TAR and MTAR Models

TAR – Explanatory Variable	**$\Delta(P/E)_t$**	**ΔP_t**	**ΔE_t**
IND{[$(P/E)_{t-1}$, P_{t-1}, or E_{t-1} – 19.70]}	-0.035	-0.3061	0.0329
(*t*-statistics)	(-6.8853)	(-2.7631)	(11.5456)
(1– IND){[$(P/E)_{t-1}$, P_{t-1}, or E_{t-1} – 19.70]}	-0.0057	-0.0681	0.0001
(*t*-statistics)	(-1.8903)	(-1.0802)	(0.0941)
Akaike Information Criterion (AIC)	2.4092	8.4822	1.1953
Schwarz Baysian Criterion (SBC)	2.4282	8.5169	1.2206
Augmentation terms - omitted	6	6(E) and 5(P)	4
Q-statistics	0.57	1.15	0.34
[Probability]	[0.44]	[0.28]	[0.56]
Attractor: Non-standard F-Test	25.12	4.40	66.66
Linearity: Standard F-Test	25.08	3.48	98.65
MTAR – Explanatory Variable	**$\Delta(P/E)_t$**	**ΔP_t**	**ΔE_t**
IND{[$(P/E)_{t-1}$, P_{t-1}, or E_{t-1} – 18.21]}	-0.0023	-0.0642	0.0202
(*t*-statistics)	(-0.5619)	(-0.7749)	(9.4187)
(1– IND){[$(P/E)_{t-1}$, P_{t-1}, or E_{t-1} – 18.21]}	- 0.0315	-0.1736	-0.0013
(*t*- statistics)	(-7.3094)	(-1.9442)	(-0.54)
Akaike Information Criterion (AIC)	2.4558	8.4658	1.2090
Schwarz Baysian Criterion (SBC)	2.4779	8.5037	1.2407
Augmentation terms - omitted	6	7(E) and 6(P)	4(E) and7(P)
Q-statistics	2.50	3.45	2.15
[Probability]	[0.29]	[0.18]	[0.83]
Attractor: Non-standard F-Test	26.83	2.19	44.59
Linearity: Standard F-Test	23.97	0.81	46.42

Notes: Numbers in parentheses are the estimated *t*-values and unless otherwise specified, the significance level is assumed to be 5 percent. Numbers in square brackets are the estimated probability. To correct for autocorrelation, different lagged residuals are deemed necessary in the six models reported.

During prosperous times in which $(P/E)_{t-1} \geq \Gamma$, the TAR model is significantly capable of closing down the discrepancy between $(P/E)_{t-1}$ and the long-run equilibrium (Γ) at the rate of 3.5 percent on a monthly basis (42 percent annually). The driving force in this case appears to be an increase in ΔE, while ΔP is noticeably falling. However, during the precipitating downswing $(P/E)_{t-1} < \Gamma$, the speed of adjustment is negligible though significant at the 5 percent level, to which ΔP and ΔE do not contribute anything noteworthy. Overall, the price in both scenarios is unstable and unreliable, and the P/E ratio in recession inherits very little from the random walk property of stock prices in the numerator. Both the composite price index and respective earnings lose their rights to speak for the volatility of P/E below the threshold. Furthermore, the null hypotheses of the lack of an attractor and linearity are rejected at any significance level, but accepted simultaneously for the composite stock price index.[8] Those two significant F-tests demonstrate the presence of non-linear dynamic adjustments in the P/E ratio time series, and illustrate the persistence of the P/E ratio in the high-value stage, which is mostly determined by the contemporary annualized earnings. Based on the high probabilities of the estimated Q-statistics, the autocorrelation problem has been taken care of at the 5 percent level in the estimated NLECMs.

The estimated threshold for the MTAR model using the same procedure as mentioned before is $\Gamma = 18.21$. The findings of the MTAR model, as can be seen in the lower portion of Table 3, are noticeably different from those reported for the TAR model. In this scenario, during a recessionary period, $\Delta(P/E)_t$ tends to decrease in response to $(P/E)_{t-1} < \Gamma$ significantly at the rate of about 3.1 percent per month (37.2 percent annually). However, there is no significant speed of adjustment when $(P/E)_{t-1} > \Gamma$ even at the 10 percent significance level, while ΔE is significantly rising. Note also that both the estimated AIC and SBC for the MTAR model in which P/E can have different rates of autoregressive decaying, are slightly larger than those of the TAR model. Moreover, the MTAR modeling comes in handy since the P/E time series purportedly has a tendency (momentum) to move more in one direction than the other. In short, since the exact nature of the apparent non-linearity has not been determined *a priori*, the estimated TAR is "marginally" the preferred model. As was the case with TAR, the null hypotheses of both linearity and the absence of an attractor for the P/E ratio and E are decisively rejected at any significance level.[9] Additionally, the reported MTAR models appear to be devoid of significant residual autocorrelation at the 5 percent level.

IV. Concluding Remarks

This paper reexamines the stationarity (the mean reversion property) of the P/E ratio time series, which has profound implications for the efficient market hypothesis. The issue has been explored by employing the Fourier approximation. The Fourier unit root test is capable of flexibly keeping track of potential breakpoints embedded in the P/E time series. The findings suggest that the P/E ratio persists in the long run and unquestionably has an affinity for the long-run equilibrium. To explore this matter further, the estimated P/E thresholds are incorporated in each of the two non-linear error correction models. The TAR and MTAR models agree with the verdict that when the P/E ratio stays afloat in the higher/lower value regime respectively, the stationary process of P/E is confirmed. Indeed, the P/E ratio eventually heads back to its historical mean value (threshold). However, in the lower value regime, the "marginally" preferred TAR model shows that the P/E ratio series has very little tendency for approaching its long-run destination.

In line with the findings presented in this study, the mean reversion property of the P/E ratio (extensively reported in the finance and economic literature), is justifiable if its ingredients (price and monthly earnings) display such a tendency. With that in mind, both TAR (below its long-run equilibrium) and MTAR (above its long-run equilibrium) models provide empirical evidence suggesting that the P/E ratio is indeed mean averting, to which the stock price and monthly earnings donate absolutely nothing. Consequently, solid empirical evidence for the mean reverting characteristic of the P/E ratio with its profound market efficiency implications remains elusive.

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[8] For over 250 observations and 4 augmentation terms, the critical values for the TAR model are 6.29, 7.15, and 8.35 at the significance level of 5, 2.5 and 1 percent, respectively, see Enders (2010), p. 494.

[9] The critical values of $\phi_{m,}$ for the MTAR model with more than 250 observations and 4 augmentation terms are 5.54, 6.39, and 7.61 at the significance level of 5, 2.5 and 1 percent, respectively. See Enders (2010), p. 494.

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