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Two New Sequencing Rules for the Non-Preemptive Single Machine Scheduling Problem

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In this paper, two new job sequencing rules are introduced for the non-preemptive single machine scheduling problem. Through a simulation study, these new rules are compared to First Come - First Served, Shortest Process Time, Earliest Due Date, Critical Ratio, and Shortest Slack sequencing rules. The rules are compared based on five performance criteria of average delay, average flow time, number of delayed jobs, longest delay, and average total of earliness and delay. Simulation results show that the new rules are promising and effective.

Keywords: Job Sequencing, Sequencing Rules, Single Machine Scheduling

JEL Classification: D02, D24, D29

I. Introduction

The non-preemptive single machine scheduling problem deals with sequencing *n* independent jobs to be processed by one machine. Jobs have to be performed sequentially and cannot be processed simultaneously. Jobs cannot be preempted; once a job starts, it has to finish before another job can start.

The objective of this scheduling problem is to find the optimal sequence that minimizes a performance metric such as average delay, average flow time, number of delayed jobs, longest delay, or average total of earliness and delay. For a problem with *n* jobs, there are *n!* distinct sequences (Baker and Trietsch, 2013). Therefore, this scheduling problem becomes very complex for large *n*’s. As the number of jobs, *n*, increases, the number of distinct sequences, *n!*, increases exponentially. In other words, finding the optimal sequence is a difficult, and time consuming, task for problems with large *n*’s. Hence, sequencing rules have been developed to tackle the problem. Although these sequencing rules do not necessarily generate the optimal solution, they aim to find high-quality solutions in a very short amount of time. The well-known sequencing rules for the non-preemptive single machine scheduling problem are as follows (Pinedo and Seshadri, 2001; T'Kindt and Billaut, 2006; Brucker, 2007; Pinedo, 2009; Pinedo, 2012; Jacobs and Chase, 2013; Stevenson, 2014; Reid and Sanders, 2015; Cachon and Terwiesch, 2016; and Heizer *et*. *al*., 2016):

(1) First Come - First Served (FCFS): Jobs are sequenced according to arrival time – earliest arrival time first.
(2) Shortest Process Time (SPT): Jobs are sequenced according to process time – shortest process time first.
(3) Earliest Due Date (EDD): Jobs are sequenced according to due date – earliest due date first.

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(4) Critical Ratio (CR): Jobs are sequenced according to ratio of time remaining until due date to process time – lowest ratio first.
(5) Shortest Slack (SS): Jobs are sequenced according to slack (time remaining until due date minus process time) – shortest slack first.

The motivation of this study has been to find more effective sequencing rules. In this paper, two new sequencing rules are introduced. Through a simulation study, these new proposed rules are compared with the above sequencing rules based on five performance criteria of average delay, average flow time, number of delayed jobs, longest delay, and average total of earliness and delay. The results show that the proposed sequencing rules are overall more effective and generate better job sequences. However, there are no studies in the literature that have presented these two sequencing rules. These new sequencing rules can specifically help businesses minimize order (job) delays. Minimizing order delays increases customer satisfaction and improves the image and reputation of the company. In long term, this leads to attracting more customers and increasing revenue and profits for the business.

In Sections 2 and 3 of this paper, the first and second proposed sequencing rules are presented respectively and the comparison results are discussed. Conclusions are presented in Section 4.

II. First Proposed Sequencing Rule

SPT and EDD sequencing rules are more promising and overall generate better sequences than FCFS, CR, and SS. This can be seen later in this paper where the rules are compared based on five performance measures of average delay, average flow time, number of delayed jobs, longest delay, and average total of earliness and delay. The idea explored in this paper is combining SPT and EDD rules to achieve even better sequences. When SPT is used, process times are considered to sequence jobs and due dates are ignored. When EDD is used, due dates are considered and process times are ignored. To consider both parameters, the Process time and Due date Total (PDT) rule is introduced in this paper as the first proposed sequencing rule. In this new rule, jobs are sequenced according to the total of process time and due date (days from now), the smallest total first. Although the total of process time and due date does not have a particular meaning, this rule simultaneously takes both process time and due date into account to sequence jobs.

To evaluate the effectiveness of PDT, this proposed rule is compared with other sequencing rules in a simulation study. In this simulation study, 10,000 different problems are generated randomly. Each problem has 10 jobs. For each job, the job arrival time, the job process time, and its due date are generated from the uniform distribution. Arrival times are drawn from a uniform distribution on the interval of 0 to 15 days ago, process times on the interval of 1 to 15 days, and due dates (days from arrival time) on the interval of 3 times the process time and 60 days. For each randomly generated problem, jobs are sequenced using the five traditional sequencing rules as well as the proposed sequencing rule. In other words, for each problem six sequences of jobs have been created. The problems are generated and the sequencing rules are coded in MATLAB.

The sequencing rules are compared based on five criteria or performance measures (Chen *et al.*, 1999; Pindeo and Seshadri, 2001; T'Kindt and Billaut, 2006; Brucker, 2007; Pinedo, 2009; Pinedo, 2012; Baker and Trietsch, 2013; Reid and Sanders, 2015; Cachon and Terwiesch, 2016; and Heizer *et al.*, 2016):

(1) Average delay
(2) Average flow time: Flow time is the time a job spends in the system or the time a customer has to wait.
(3) Number of delayed jobs
(4) Longest delay
(5) Average total of earliness and delay

A sample problem solved based on EDD is shown in Table 1. In Table 1, jobs are sequenced based on due dates. The description of each column is as follows:

(1) Arrival time (days ago): The arrival time of 0 means today and the arrival time of -3 means 3 days ago.
(2) Process time (in days)
(3) Due date (days from now)
(4) Finish Time (days from now): Finish time of the previous job + (2)
(5) Delay: Max ((4) – (3),0)
(6) Flow time: (4) – (1)
(7) Earliness: Max ((3) – (4),0)
(8) Total of earliness and delay: (5) + (7)

Table 1: A Solved Problem Based on EDD

Job	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
7	-14	3	1	3	2	17	0	2
4	-12	1	16	4	0	16	12	12
8	-8	10	22	14	0	22	8	8
1	-3	9	31	23	0	26	8	8
10	-2	4	34	27	0	29	7	7
9	-3	9	39	36	0	39	3	3
6	-3	14	42	50	8	53	0	8
2	-10	5	43	55	12	65	0	12
3	0	15	50	70	20	70	0	20
5	-2	8	54	78	24	80	0	24

For the EDD sequence shown in Table 1, the average delay is 6.6 days, the average flow time is 41.7 days, the number of delayed jobs is 5, the longest delay is 24 days, and the average total of earliness and delay is 10.4 days.

The above sample problem is solved based on PDT, the proposed sequencing rule in this paper. The results are presented in Table 2. In Table 2, jobs are sequenced based on the total of process time and due date. The total is shown in column (9).

Table 2: A Solved Problem Based on PDT

Job	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
7	-14	3	1	3	2	17	0	2	4
4	-12	1	16	4	0	16	12	12	17
8	-8	10	22	14	0	22	8	8	32
10	-2	4	34	18	0	20	16	16	38
1	-3	9	31	27	0	30	4	4	40
9	-3	9	39	36	0	39	3	3	48
2	-10	5	43	41	0	51	2	2	48
6	-3	14	42	55	13	58	0	13	56
5	-2	8	54	63	9	65	0	9	62
3	0	15	50	78	28	78	0	28	65

For the PDT sequence shown in Table 2, the average delay is 5.2 days, the average flow time is 39.6 days, the number of delayed jobs is 4, the longest delay is 28 days, and the average total of earliness and delay is 9.7 days.

Simulation results are presented in Table 3, Figure 1, Figure 2, Figure 3, Figure 4, and Figure 5, which are MATLAB outputs. The numbers are the averages for the 10,000 randomly generated problems. As seen, PDT has generated the lowest average delay and outperforms all sequencing rules. PDT has generated the second-lowest average flow time after SPT. As mentioned by Pinedo and Seshadri (2001), T'Kindt and Billaut (2006), and Pinedo (2012), SPT provides the optimal sequence with regard to the flow time criterion. PDT has generated the second-lowest number of delayed jobs slightly more than SPT. PDT has generated the second-lowest longest delay after EDD. EDD provides the optimal sequence with respect to the longest delay criterion (Pinedo and Seshadri 2001; T'Kindt and Billaut 2006; and Pinedo 2009). Also, PDT has generated the third-lowest average total of earliness and delay, after CR and EDD.

Table 3: Simulation Results

Criterion	FCFS	SPT	EDD	CR	SS	PDT
Average delay	15.1	9.1	8.9	9.8	11.0	7.9
Average flow time	51.5	40.4	46.7	49.4	49.9	44.1
Number of delayed jobs	6.2	4.4	5.2	7.1	5.9	4.6
Longest delay	45.8	37.5	28.6	29.8	30.8	29.7
Average total of earliness and delay	20.8	19.8	13.1	12.2	14.1	13.7

Figure 1: Average Delay

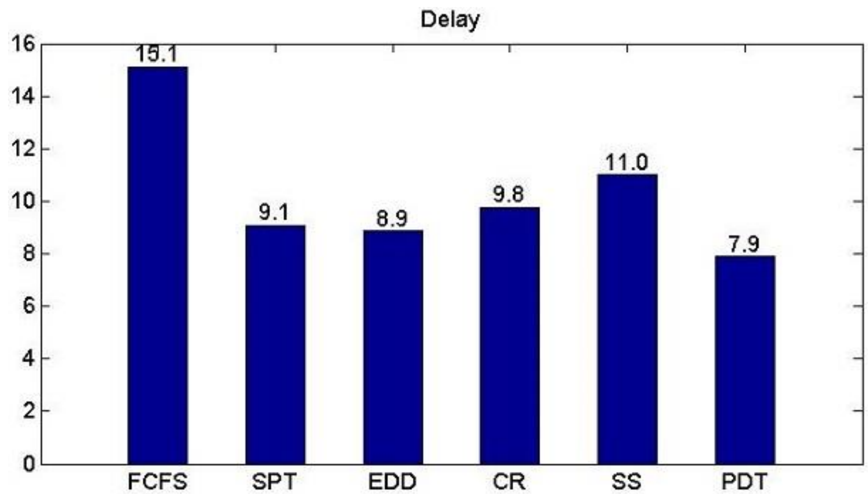


Figure 2: Average Flow Time

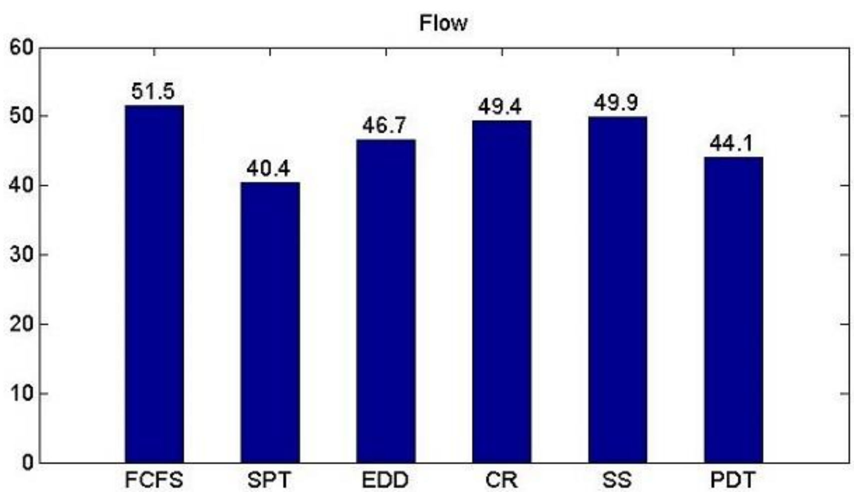


Figure 3: Number of Delayed Jobs

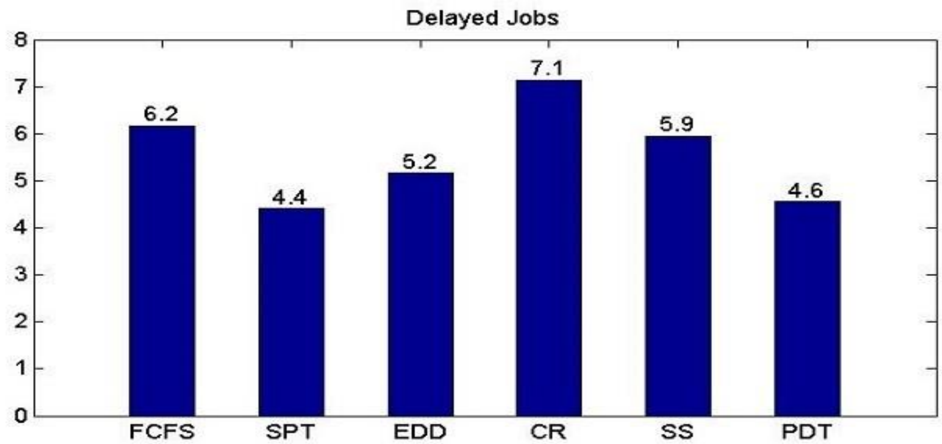


Figure 4: Longest Delay

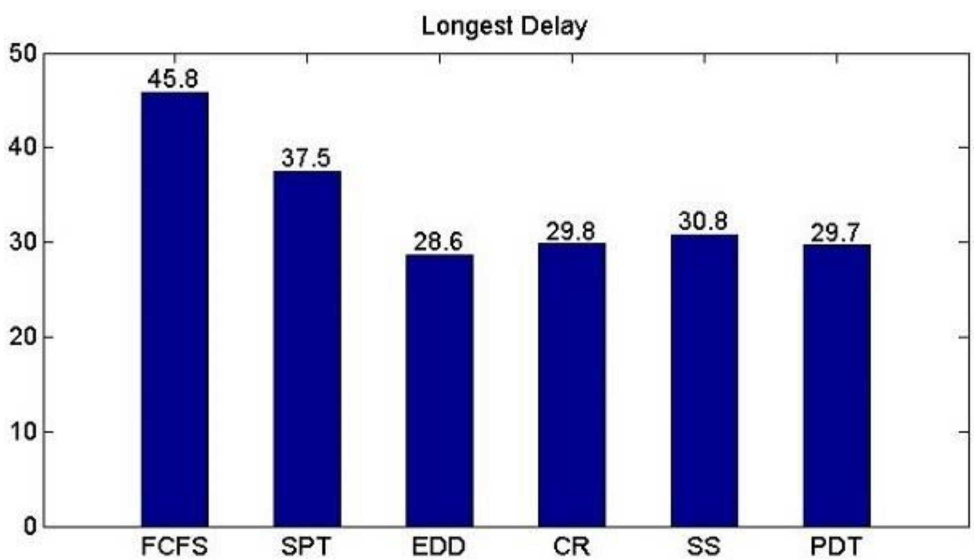
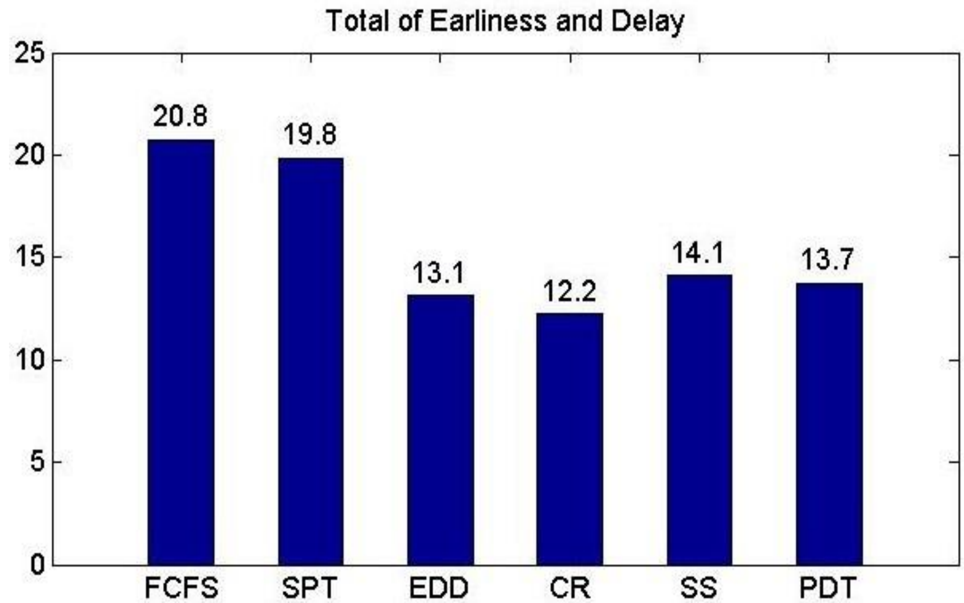


Figure 5: Average Total of Earliness and Delay



As seen in Table 3, Figure 1, Figure 2, Figure 3, Figure 4, and Figure 5, PDT has outperformed FCFS and SS with respect to all five criteria. PDT has performed better than CR with regard to the four criteria of average delay, average flow time, number of delayed jobs, and longest delay. PDT has provided better results than SPT with regard to the three performance measures of average delay, longest delay, and average total of earliness and delay. Also, PDT has performed better than EDD in terms of the three criteria of average delay, average flow time, and number of delayed jobs.

Table 4 shows the rank of each sequencing rule with regard to each performance measure. As seen, PDT and EDD are the only sequencing rules that do not rank worse than third. Overall, PDT has the best average rank, followed by EDD and SPT.

Table 4: Ranks of Sequencing Rules

Criterion	FCFS	SPT	EDD	CR	SS	PDT
Average delay	6th	3rd	2nd	4th	5th	1st
Average flow time	6th	1st	3rd	4th	5th	2nd
Number of delayed jobs	5th	1st	3rd	6th	4th	2nd
Longest delay	6th	5th	1st	3rd	4th	2nd
Average total of earliness and delay	6th	5th	2nd	1st	4th	3rd
Average Rank	**5.8**	**3**	**2.2**	**3.6**	**4.4**	**2**

Table 5 shows the deviation (in percent) of the result of each sequencing rule from the result of the best sequencing rule with respect to each criterion. As seen, results of PDT are very close to the best results and have small deviations. PDT is the only sequencing rule that does not deviate from the best sequencing rule more than 12%. Overall, PDT has the lowest average deviation, followed by EDD and SPT. As observed, the average deviation for PDT is in the single digits and significantly less than the third-lowest average deviation. All in all, the simulation results presented in this section show that PDT is a very effective and promising sequencing rule.

Table 5: Deviation from the Best Result

Criterion	FCFS	SPT	EDD	CR	SS	PDT
Average delay	91%	15%	13%	24%	39%	0%
Average flow time	27%	0%	16%	22%	24%	9%
Number of delayed jobs	41%	0%	18%	61%	34%	5%
Longest delay	60%	31%	0%	4%	8%	4%
Average total of earliness and delay	70%	62%	7%	0%	16%	12%
Average Deviation	**58.0%**	**21.7%**	**10.8%**	**22.4%**	**24.0%**	**6.0%**

III. Second Proposed Sequencing Rule

Another way to combine SPT and EDD is to sequence jobs based on the weighted total of process time and due date. This new proposed rule is called Process time and Due date Weighted Total (PDWT) in this paper. The weighted total is calculated based on the following formula:

$$PDWT = w * PT + (1-w) * DD$$

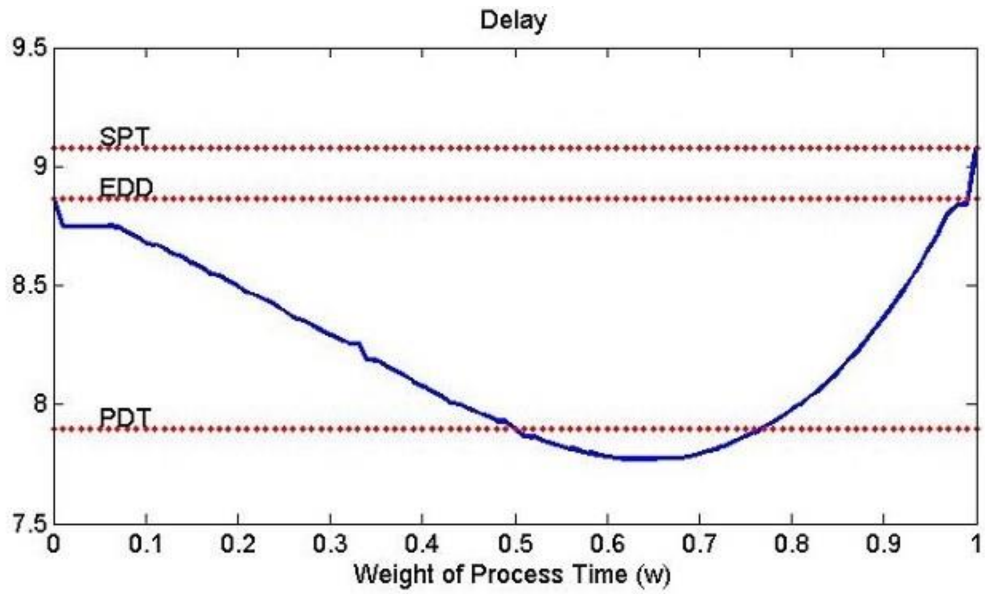
where PT is process time, DD is due date (days from now), and *w* is a real number between 0 and 1. In the above equation, *w* is the weight of process time and *1-w* is the weight of due date.

In a simulation study, PDWT is compared with SPT, EDD, PDT, and other sequencing rules ranking third or better with respect to each performance measure. In this analysis, *w* varies from 0 to 1 with an increment of 0.01 (i.e. 0.00, 0.01, 0.02, … , 0.98, 0.99, 1.00). Obviously, *w* of 0 represents EDD rule, *w* of 1 represents SPT rule, and *w* of 0.5 represents PDT rule as in PDT rule the weights of process time and due date are equal. For each value of *w*, the 10,000 randomly generated problems have been solved, and the average results are shown in Figure 6, Figure 7, Figure 8, Figure 9, and Figure 10. In these figures, the solid curve is the result of PDWT for various *w's*. Also, the results of PDT, SPT, and EDD along with the results of other sequencing rules ranking third or better with regard to each criterion are depicted in the figures.

The results of average delay are presented in Figure 6. As mentioned before, PDT has generated a lower average delay than EDD and SPT. As seen in Figure 6, PDWT generates lower average delays than PDT for certain weights, *w's*. Through numerous simulation runs, PDWT was observed to always perform better than PDT for weights of 0.51 to 0.76. Generally, weights of 0.61 to 0.67 generated the lowest average delays. Specifically, weights of 0.63, 0.64, and 0.65

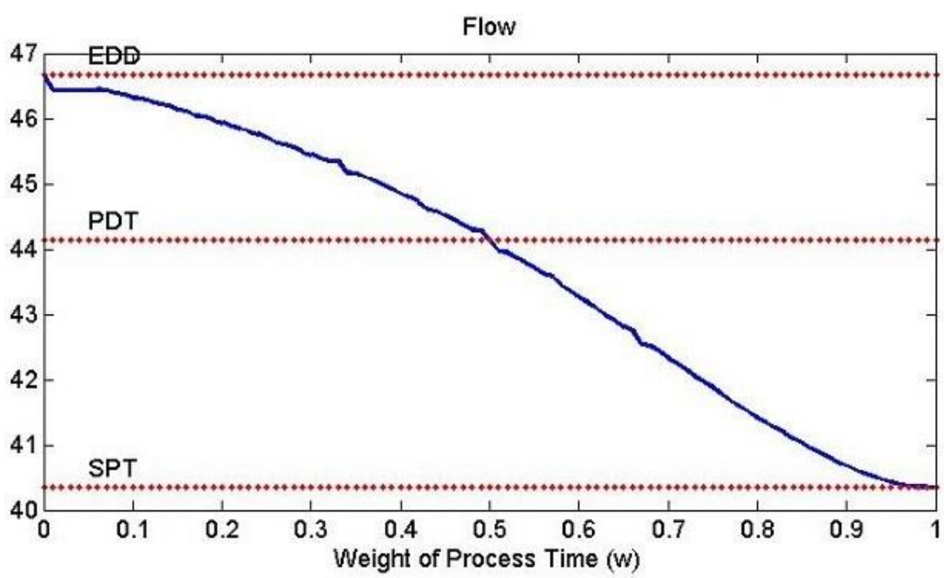
resulted in the three lowest average delays. Weight of 0.64 generated the lowest average delay in most runs, and 0.63 resulted in the lowest average delay occasionally.

Figure 6: Average Delay for PDWT



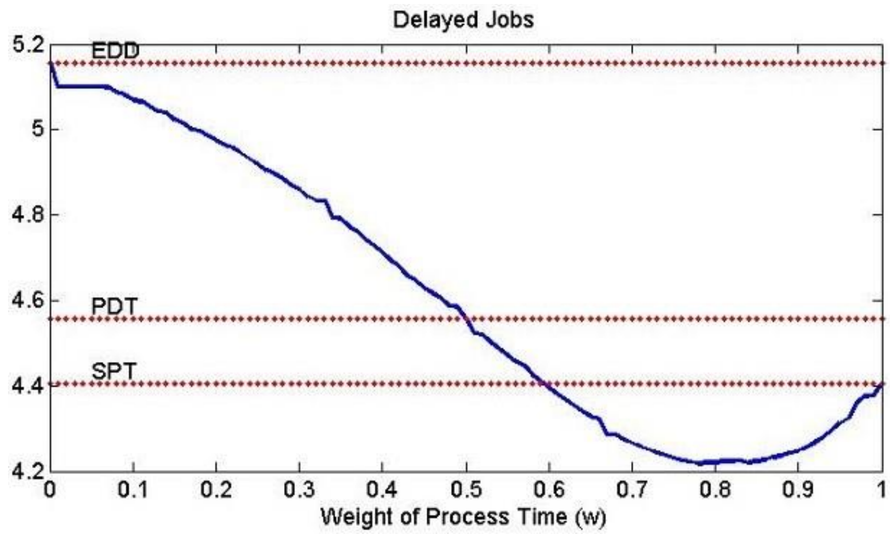
The results of average flow time are shown in Figure 7. As mentioned before, SPT minimizes the average flow time. As shown, PDWT performs better than PDT if *w* is more than 0.50.

Figure 7: Average Flow Time for PDWT



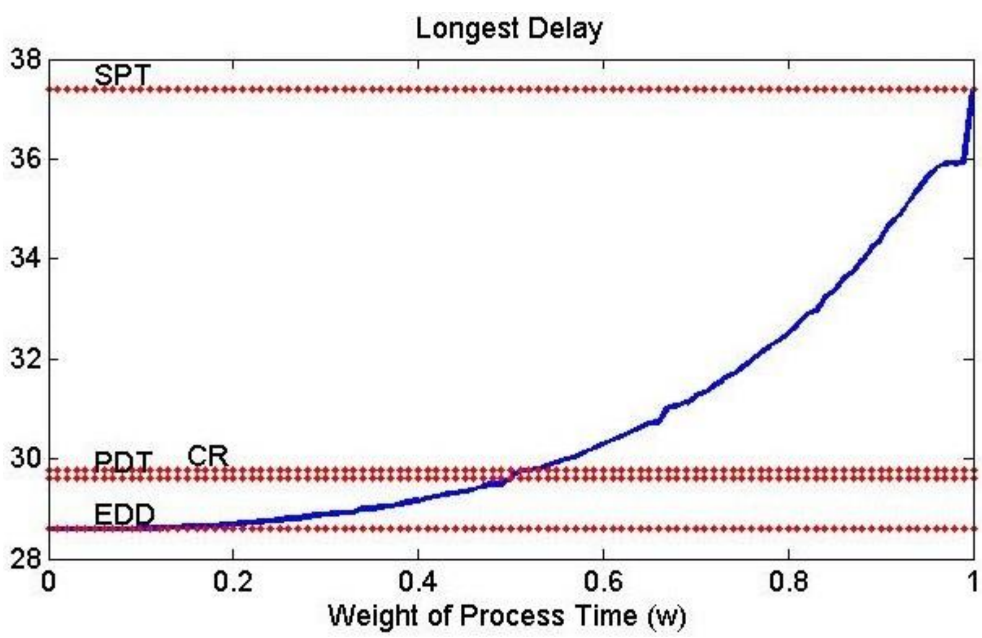
The results of the number of delayed jobs are presented in Figure 8. As mentioned before, while PDT performs better than EDD, SPT is slightly better than PDT. However, as seen in Figure 8, PDWT can perform better than both PDT and SPT. As shown, PDWT is better than PDT for weights more than 0.50. Through numerous simulation runs, PDWT was observed to always perform better than SPT for weights of 0.60 to 0.99. Generally, weights of 0.77 to 0.85 generated the lowest numbers of delayed jobs. Weights of 0.78 to 0.82 alternatively resulted in the lowest number of delayed jobs in different runs.

Figure 8: Number of Delayed Jobs for PDWT



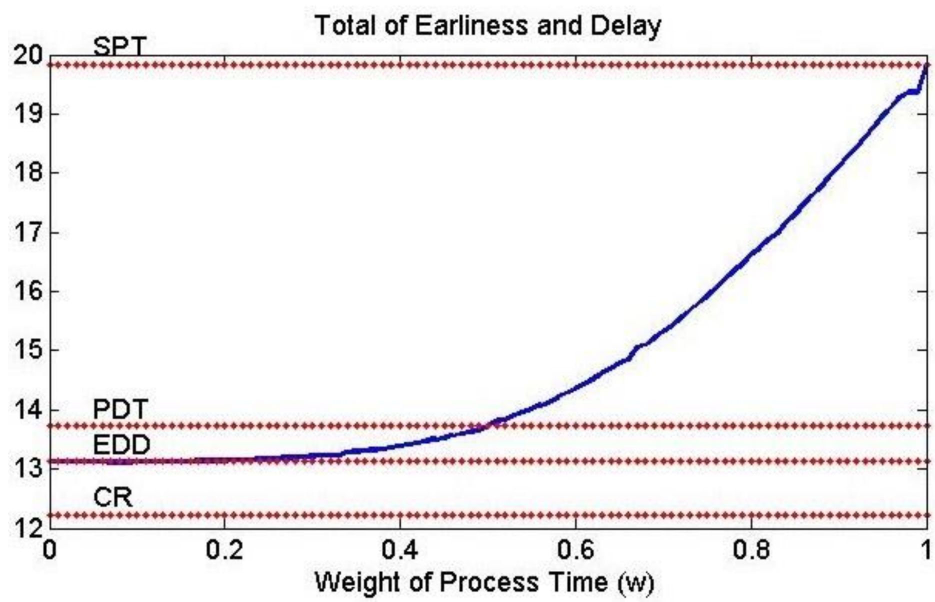
The results of longest delay are shown in Figure 9. As mentioned before, EDD minimizes the average flow time. As shown, PDWT performs better than PDT if the weight is less than 0.50.

Figure 9: Longest Delay for PDWT



The results of average total of earliness and delay are shown in Figure 10. As mentioned before, CR has generated the lowest average, and EDD performs better than PDT. As seen, PDWT performs better than PDT if the weight is less than 0.50. Through numerous simulation runs, PDWT and EDD results were observed to be very close for weights less than 0.20. Interestingly, PDWT performs slightly better than EDD and generates lower averages usually for weights of 0.01 to 0.11.

Figure 10: Average Total of Earliness and Delay for PDWT



IV. Conclusions

The non-preemptive single machine scheduling problem is complex for large-size instances. Two new job sequencing rules of PDT and PDWT are proposed in this paper. These rules are developed based on the combination of process time and due date. A simulation study has been performed to compare these new rules with five well-known sequencing rules. Simulation results show the effectiveness of PDT and PDWT. Based on comparison over five performance measures, PDT has the best average rank and the lowest average deviation from the best result. Specifically, PDT ranks first with regard to average delay, and PDWT generates lower average delays for certain weights. Additionally, PDWT performs better than all sequencing rules with regard to the number of delayed jobs for certain weights.

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